OBSERVING MATHEMATICAL PROBLEM SOLVING: PERSPECTIVES ON STRUCTURED, TASK-BASED INTERVIEWS

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Structured, task-based individual interviews are one aspect of an ongoing mathematics education research study of elementary school children at Rutgers University. The goals of the interviews are to observe complex, mathematical problem-solving behavior in detail, and to draw inferences from these observations about the children's thinking and development. This paper discusses the scientific underpinnings of the methodology, the role of cognitive theory in structuring an interview, constraints and limitations imposed by the social/psychological context of an interview, and the interplay between task variables, observed behaviors, and inferred cognitions. Some principles of interview design and construction are suggested for consideration by the mathematics education research community.

The structured clinical interview is finding increasing acceptance today as a research method for the study of mathematical learning and problem solving. In general, such structured interviews are for the twin purposes of (a) *observing* mathematical behavior, usually in a problem-solving context, and (b) *drawing inferences* from the observations about the problem solver's cognitions and/or affect. The ideas in the present paper grew out of several earlier studies making use of task-based, individual interviews (Goldin, 1985, 1986; Goldin and Landis, 1985, 1986; Bodner and Goldin, 1991a,b; DeBellis and Goldin, 1991). Presently a group of us at Rutgers University are developing a series of such interviews in the context of a three-year, longitudinal study of individual children's mathematical development (Goldin, DeBellis, DeWindt-King, Passantino, and Zang, 1993). The perspectives presented here are helping to shape this development; thus comment and criticism are invited.

Adoption of the task-based interview as a principal research tool in our study raises a series of questions: (1) In what sense do such interviews permit a *scientific* investigation? What are the implications concerning replicability of results, comparability of outcomes, or generalizability from observations? (2) What is the role of *theory* in structuring an interview? To what extent are the observations made contingent on (tacit or explicit) theoretical assumptions underlying the interview? How does theory guide the drawing of inferences about cognition and/or affect from these observations? What is the interplay between *task variables*, observed behaviors, and the inferences one can draw? (3) What constraints or limitations are imposed by the social and psycho- logical *context* of the interview? (4) What general *principles* of interview design and construction, if any, are appropriate for the mathematics education research community to adopt? Is it possible through such principles to optimize the information gathered through a task-based interview?

The intent of this paper is to raise these questions for discussion, to offer a few illustrative examples from the study currently in progress, and to propose some preliminary and partial answers for consideration.

A RESEARCH STUDY IN PROGRESS

In the study now under way, the mathematical development of an initial group of 22 children is being observed over three years; we hope half or more of the children will remain in the study for the full term. Subjects at the outset, in 1992-93, were in the third and fourth grades (ages 8 to 10) in a cross-section of schools in New

Jersey. Their teachers are participants in a mathematics education reform partnership (MaPS: Mathematics Projects in Schools) sponsored by the Rutgers Center for Mathematics, Science, and Computer Education, and directed by Carolyn Maher and Robert Davis. One component of the study consists of six task-based, individual inverviews with each child over the three years, during which the children's problem-solving is videotaped.

The purpose of the study is thus purely *exploratory* and *descriptive*--subjects are not a random sample, and no general hypotheses are being tested. Overall we hope to describe individual mathematical development in as much detail as possible, focusing not on standard, discrete skills or algorithmic problem solving, but on the growth of complex, internal representational capabilities. The framework for describing these capabilities is based on a model for mathematical problem-solving competency embodying five kinds of systems of internal, cognitive representation (Goldin, 1987; 1992): (a) a verbal/syntactic system (use of language); (b) imagistic systems (visual/spatial, auditory, kinesthetic encoding), (c) formal notational systems (use of mathematical notation), (d) planning, monitoring, and executive control (use of heuristic strategies), and (e) affective representation (changing moods and emotions during problem solving). Of particular interest are interactions among these processes, and the interplay between the children's (internal) representations and external representations that they use or construct during the interviews.

Since the study is longitudinal, a major focus is how over a period of time systems of representation *develop* in the child. In this respect the theoretical model incorporates three main stages: (a) an inventive/semiotic stage, in which internal configurations are first assigned "meaning", (b) a period of structural development, driven by the meanings first assigned, and (c) an autonomous stage, in which the representational system functions flexibly and in new contexts.

Though the analysis of outcomes is theoretically-based, we seek not only to observe and draw inferences from *expected* processes, but also to search for unanticipated occurrences. The hoped-for results include: (1) a set of detailed, descriptive case studies of individual mathematical development, with accompanying videotapes, protocols, analyses and interpretations; (2) improved capability for observing and drawing inferences from mathematical behavior; (3) further refinement and development of the theoretical model for problem solving, including identification of inadequancies, and progress toward an assessment framework; and (4) conjectures for wider investigation through experimental studies.

Design of the interviews themselves is tied to these goals. It includes the following steps: (a) planning in relation to mathematical content and strucutre, anticipated observations, and inferences; (b) the creation and critique of an interview script; (c) pilot-testing and revision of the script; and (d) rehearsal and training of clinicians. As this is written, two of the six interviews have been completed (Spring 1992 and Fall 1993) and the third is under design to be administered next month (Spring 1993). The following are some elements of the interview scripts (see Goldin *et al.,* 1993 for more detail; the full scripts as they are completed are available from the authors):

Interview #1 [55 pages]: An additive structure is embodied in an arithmetic sequence, represented *via* a geometric arrangement of dots. The first three cards in the sequence are presented: "Here is the first card, here is the second card, and here is the third card."

A series of exploratory questions follows, with contingencies based on the nature of the child's responses; special emphasis is placed on exploring the student's pattern-construction and use of external representations.

Interview #2 [38 pages]: A series of questions explores "one half" and "one third" in many embodiments. "When you think of 'one half' [subsequently, when you think of 'one third'], what comes to mind?" Included are requests to take one half and one third of a number of objects (12 apples), various two-dimensional shapes (square, circle, six-petaled flower), an array, and a solid wooden cube. Then the child is asked to think about painting the cube red and cutting it in a number of different ways, describing the pieces that would result. A multiplicative structure is embodied in cutting across different dimensions. Special emphasis in this interview is placed on exploring visualization by the child.

Interview #3 (under design): Two different problems are presented successively: (1) cutting a birthday cake (without or with frosting) to share equally among two or three children, and (2) a problem about moving colored M&M candies back and forth between two jars. Both problems embody symmetry and coordination of conditions--the first in the context of volume and area, the second in a numerical context. Emphasis is placed on exploring the child's affect, as well as metacognitions about the two tasks.

Interviews are planned to take approximately 45 minutes (less than one class period). In all three interviews, alternative embodiments for external representation are provided: paper and pencil, markers, cards, chips and/or other manipulatives, paper cut-outs, etc., in accordance with the task. The questions tend to increase in difficulty, so that each child begins with a level of comfort, but even mathematically advanced children encounter some questions that are challenging before the interview ends. Free problem solving is encouraged wherever possible, with (specified) hints given or suggestions made only after the child has had the opportunity to respond spontaneously. All responses are accepted by the clinician (with occasional, specified exceptions), with "wrong" and "correct" answers treated similarly. Follow-up questions are without overt indication of the correctness of earlier responses.

Two videocameras are in operation simultaneously during each interview--one focusing on the clinician and the child, and the second focusing on the student's work with his or her hands (paper and pencil, or manipulatives). An observer also makes notes during the interview on a copy of the script. Subsequently the videotapes are transcribed, viewed, and analyzed.

The latter three interviews remain to be developed for administration during 1993 and 1994. Interview #4 will place special emphasis on exploring the child's strategic and heuristic thinking. In interview #5 we plan to include an interactive computer environment. Interview #6 will return to selected mathematical ideas from the earlier interviews.

As this is written the first two sets of interviews are being transcribed, and analysis is commencing. The purpose of this paper is not to report on the outcomes to date, but to discuss general perspectives on structured, task-based interviews of this sort, and to invite comment.

ON THE SCIENTIFIC NATURE OF TASK-BASED INTERVIEWS

This study is exploratory. Consisting as it does of a collection of individual case studies, its outcomes are not scientifically reproducible. Nevertheless we have devoted great effort to structuring the interviews ahead of time to be both *flexible* and *reproducible*. Flexibility by the clinician in such an interview is essential to allow for the enormous differences that occur in individual problem-solving behaviors, and (since a major goal is to observe processes the child uses spontaneously) to avoid "leading" the child in a predetermined direction.

Reproducibility, however, means that the clinician is not merely inventing questions as the child responds. It permits, to a certain imperfect degree, the "same" interview to be administered by different clinicians, to different children, in different contexts. To accomplish this sufficiently many contingencies must be anticipated, and the criteria for the clinician's choices of questions or suggestions must be made explicit *in advance* for each contingency. This is what we have sought to do in the process of interview design.

For example in interview #1, after a brief pause (to allow for the possibility of spontaneous responses to the three presented cards), the child is asked, "What do you think would be on the next card?" Contingencies then include "response" and "don't know"; if the child responds, the next contingencies include "offers a complete, coherent reason" or "has not yet given a complete, coherent reason", with or without having constructed a "coherent external representation." The definitions (from the directions in the interview #1 script) are as follows:

> "A complete and coherent verbal reason means one based on a described pattern. A coherent external representation means a drawing, picture, or chip model. It is *not* required that the 'canonical' fourth card (with 7 dots) be drawn, or the canonical pattern described, for a response to be considered a complete and coherent reason and a coherent external representation. An answer such as '7, because it's 2 more' is a coherent verbal reason, but not considered complete because it refers only to finding the next card and not to the basis for the pattern. An answer such as '7, because this card has 2 more than that one, so the next one has 2 more also' would be considered coherent and complete. If there is a discrepancy between the number of dots stated and the number in an external representation, the verbal reason is not considered 'coherent'. This is intended to describe the 'boundary' between responses that are and are not accepted as complete and coherent at this stage."

This is the level of detail at which many contingencies are considered. The clinician's next question or suggestion (e.g., "Why do you think so?" or "Can you show me what you mean?" leading if necessary to "Can you show me using some of these materials?") depends on the contingency which best describes the child's response. Such a level of description seeks to make *explicit* the usually *tacit* conditions that ordinarily influence a skilled clinician. In principle, a detailed structured interview description permits (a) *replicability* of the interview itself, though contextual and other factors may still vary from occasion to occasion; (b) *comparability* of interview outcomes between different children, among different populations of children, or across different conditions; (c) subsequent experiments to investigate the *generalizabilty* of observations made in individual case studies; (d) explicit *discussion and critique* of the contingencies, the criteria for the clinician's responses, etc.; and (e) an explicit basis for discussing the analysis of outcomes. For other perspectives, see Cobb (1986), Hart (1986), and Steffe (1991).

THE ROLE OF THEORY

The questions asked and the observations made during any scientific investigation depend heavily on the theory we bring to it. In my view, the main question is not whether theory *should* influence us in this enterprise--I would maintain that it always, inevitably does:

> ". . . perhaps the attempts to use the methods of science [in education] have failed because science has been misunderstood.
>
> In these attempts it had been assumed that science was primarily factual, that indeed it dealt almost solely in facts, that theory had no role in science. Careful observation of science reveals this to be false. It might be closer to the truth to say that 'facts'--at least interesting facts--are almost unable to exist *except in the presence of an appropriate theory* [emphasis in original]. Without an appropriate theory, one cannot even state what the 'facts' are." (Davis, 1984, p. 22)

The question pertaining to clinical interviews is the extent to which the influence of theory is tacit, through the unconscious assumptions of clinicians, researchers, and/or teachers, or explicit. Our goal in the present study is to be as explicit as possible.

The theoretical underpinnings of the series of task-based interviews include the concept of (internal) *competencies* and structures of such competencies, that develop over time in the child, and that can be *inferred* from observable behavior. The idea that competencies are encoded in several different kinds of internal representations, and that these *interact* with each other and with observable, external representations, is also fundamental. The key distinction between the child spontaneously bringing particular competencies to bear, or doing so only when prompted, is also theoretically-based: it involves the child's exercise of planning competencies to call on other competencies (verbal, imagistic, formal notational, etc.). These ideas have influenced the task-based interview development as follows: We pose tasks which permit the children tc perform at each step spontaneously. We explore not only the child's overt behavior, but the reason the child gives for taking each step. Recognizing that competency structures may be partially developed, we provide hints or heuristic suggestions when blockage occurs--this often permits the child to demonstrate competencies that otherwise he or she would never "get to". We seek information about each kind of internal representational system--thus, not satisfied with a coherent verbal explanation only, we encourage the child to construct a concrete, external representation. We in- clude a "cross section" of questions exploring visualization, affect, and strategic thinking.

The distinction between external and internal representation means we must attend carefully to both. We regard the tasks posed as *external* to individual children; as embodying syntax, content, context, and structure variables that we select when we design the interviews. In particular the mathematical structures of the tasks (semantic structures and formal structures-- additive, multiplicative, etc.) are consciously chosen. The children's behaviors then result from interactions between the task environment and their internal cognitive and affective representations.

THE ROLE OF CONTEXT

Interviews do not take place outside of a social and psychological *context*. We observe that the child's expectations of an interview are influenced by the fact that it is conducted by a relative stranger (the clinician); it takes place in school (and thus might involve some kind of test that "counts" toward an evaluation, and the tasks are likely to have "right" and "wrong" answers); it involves tasks unrelated to a goal or purpose generated by the child; it may be taking place at a moment when the child is alert, tired, hungry, distracted, or excited; and so forth. Seemingly small, contextual aspects of the tasks themselves may have important effects. For example in presenting the three cards interview #1, we permit the child to see the cards being drawn from a stack of cards in a manila envelope. From this minor contextual feature the child may infer that there is a deck of cards larger than the three that are shown, and possibly that there is a pattern in the cards. Three cards presented wholly "out of context" might not so readily elicit this expectation.

Since so much that may occur during a task-based interview is context-dependent, how can we consider what we observe to be more than accidental, one-time events? One important condition is that the constructs we *infer* from our observations be reasonably *stable* against contextual variations. Thus, while a child's behavior may vary considerably from one context to another, when we infer particular competencies or structures of competencies from the behavior (such as the ability to visualize cutting a cube across two perpendicular directions, inferred from a coherent description of the component pieces with appropriate gestures) we are inferring aspects of the child's cognition that we expect to be fairly stable. Understanding the context dependence of the interviews also means recognizing how very difficult it is to establish advance criteria for the inferences about each child's cognition and affect that we want to draw from our observations. The plan is to make the best conjectures possible, and to try to be explicit about the reasons for these conjectures (including relevant contextual factors) as they occur.

PRINCIPLES OF INTERVIEW DESIGN AND CONSTRUCTION

The above considerations lead me to formulate the following tentative and partial principles of interview design and construction, in trying to establish the strongest possible scientific foundation and maximize the information gathered through a task-based interview: (1) *Accessibility:* Interview tasks should embody mathematical ideas and structures appropriate for the subjects being interviewed--so that they are able to represent task configurations, conditions, and goals. (2) *Rich representational structure:* Tasks should embody meaningful semantic structures (imagistic level) and strategies of some complexity (planning and executive control level), as well as formal, symbolic structures (notational level). (3) Subjects should engage in *free problem solving wherever possible* to allow observation of spontaneous behaviors and reasons for spontaneous choices *prior* to offering prompts or suggestions. Providing premature guidance results in loss of information. This may mean some sacrifice of the speed with which the subject understands the problem, or progresses through it. (4) *Explicit criteria:* All major contingencies should be clearly addressed in the interview design, as explicitly as possible; but *without distinguishing "right" and "wrong" responses.* Thus all responses should be "accepted", with structured questions designed to provide subjects with opportunities to self-correct in any contingency. (5) Various external representational capabilities should be provided, allowing for *interaction* with an observable learning or problem-solving environment.

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